Moving M2 mirror without pointing offset

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Abstract

New telescopes, using active optics features and high quality tracking capabilities, require an high precision movement of the secondary mirror M2 during exposures. Moving this mirror to introduce an amount of decentering coma is one of the tasks of active optics. In this paper we show that this target is accomplished with high accuracy rotating the mirror around a point located near, but not exactly at the center of curvature of M2. Basic equations of the displacement of the barycenter of the geometrical blur, to the third order, are here developed, and a final relation to get the correct position around which the rotation is to be performed, is obtained. Such position is the harmonic mean of the radius of curvature and of the distance from the vertex of the neutral point, properly weighted. A brief discussion is given about the application of such results to typical two-mirror telescopes. Ray tracing results are compared to analytical ones in the case of the Italian National Galileo telescope, that will be equipped with an high precision M2 driving device; the close matching with the analytical calculations is demonstrated.

1 Introduction

For a number of planned telescopes active optics is an essential feature1,2,3. In such telescopes the best optical performances are obtained via a correction of aberration terms using main mirror deformations and moving the secondary mirror. In most of the planned telescopes4,5, and in the existing NTT6 the decentering coma term is controlled via movement of M2. Moreover, to gain angular resolution, most of such telescopes will be erected in sites of very best performances from the point of view of seeing7,8,9. In this framework strong constraints are imposed to the overall optical quality of the telescope. Values of the order of 0.1" are, thus, normally requested10. In this paper it is shown, on a theoretical basis, that the first-order calculation of the point around which M2 has to be rotated to obtain only coma variations and no offset effects in the pointing is not accurate enough.
A displacement of the same order of the coma aberration is in fact introduced. The correct relations to get the detailed movement needed to avoid this are here obtained and discussed. Comparison with numerical results (i.e. ray tracing calculations) is performed for the practical case of our interest: the Italian National Galileo Telescope\textsuperscript{11} (TNG).

2 Telescope parameters and coordinates

In all the text subscripts 1 and 2 mean values related to M\textsubscript{1} and M\textsubscript{2}. Absence of subscripts refers to the overall telescope.

A generic two-mirror telescope is shown in Fig.1. From the figure the adopted sign convention can be derived. Being \( f_{1} \) the focal length (at the first order) of M\textsubscript{1}, and \( f \) the overall one, the magnification factor \( m \) is defined as

\[ m = \frac{f}{f_{1}} \quad (1) \]

\( m \) is positive when M\textsubscript{2} is convex (as in a classical Cassegrain) and negative when M\textsubscript{2} is concave (as in a classical Gregorian).

The back focal distance \( \omega \) normalized to \( f_{1} \) is indicated with \( \beta \), such value is positive when the focal plane lies outside the space between M\textsubscript{1} and M\textsubscript{2}.

It is useful to introduce the dimensionless parameter \( k \), defined as

\[ k = \frac{y_{1}}{y_{2}} \quad (2) \]

where \( y_{1} \) and \( y_{2} \) are the height of the marginal rays on M\textsubscript{1} and M\textsubscript{2} respectively. It can be shown that, if the despace error is zero (i.e. if the secondary mirror is placed in such a way to compensate third order spherical aberration at the focal plane) the following relation holds

\[ k = \frac{1 + \beta}{m + 1} \quad (3) \]

Sign convention for the Angular Transverse Coma aberration (ATC) is desumed from the figure itself.

In this paper the surface mirror is described by

\[ z = \frac{R}{1 + K} \left[ 1 - \left( 1 - \frac{\rho^{2}}{R^{2}} (1 + K) \right)^{1/2} \right] \approx \frac{\rho^{2}}{2R} + (1 + K) \frac{\rho^{4}}{8R^{3}} \quad (4) \]

Such a surface is a sphere when \( K = 0 \), a paraboloid when \( K = -1 \). The case of a hyperboloid is described by any \( K < -1 \). The radius of the secondary mirror is related with a simple equation to the parameters of the telescope, i.e.

\[ R_{2} = \frac{2mk}{(m - 1)f_{1}} \quad (5) \]
3 Effects of M2 misalignment

The various effects of the M2 misalignment are here developed to the third-order. In subsection 3.1 the tilt-term derived from a rough first-order analysis is obtained. This is the usual approach. In subsection 3.2 the relationship between misalignment and coma term is recalled. The useful concept of neutral point is indicated. In subsection 3.3 it is pointed out that the asymmetric shape of the coma blur translates into an additional tilt term effect. Finally in subsection 3.4 the detailed, third-order approach, calculation of tilt term due to M2 misalignment is obtained.

3.1 First order tilt-term effect

A pure tilt α of M2 (around its vertex) produces an angular displacement of a paraxial ray of 2α, i.e. a linear displacement Δα on the focal plane (see Fig.1) of

$$\Delta_\alpha = 2\alpha \cdot mkf_1$$

such displacement is equivalent on the sky to an angular one
\[
\theta_\alpha = \frac{\Delta_\alpha}{f} = 2\alpha \frac{1+\beta}{m+1} = \alpha \cdot c_\alpha
\]  

(7)

that fully defines the \( c_\alpha \) coefficient. Using first-order approach, the M2 surface can be treated like a sphere. In such a condition, be \( c_l \) the coefficient such that

\[
\theta_l = l \cdot c_l
\]

(8)

the pair of relations

\[
\begin{align*}
\theta_l &= -\theta_\alpha \\
R_2 \alpha &= l
\end{align*}
\]

(9)

have to be met. This expresses analytically the observation that a rotation around the center of curvature of a sphere doesn’t produce any offset, and leads to the relationship

\[
c_l = -\frac{c_\alpha}{R_2}
\]

(10)

the overall first-order tilt-term is described by

\[
\theta = \left(\alpha - \frac{l}{R_2}\right) \frac{2(1+\beta)}{m+1} = \left(\alpha - \frac{l}{R_2}\right) \cdot c_\alpha
\]

(11)

### 3.2 Third order coma effect

Using the given sign convention and developing the fundamental relationships that link coma to misalignment of M2, it can be shown\textsuperscript{12} that

\[
\begin{align*}
\text{ATC}_l &= \left(\frac{l}{f}\right)^2 \frac{m-1}{32F^2} \left[K_2 - \left(\frac{m+1}{m-1}\right)\right] = q_l \cdot l \\
\text{ATC}_\alpha &= \alpha^3 \left(\frac{m-1}{16F^2}\right) = q_\alpha \cdot \alpha
\end{align*}
\]

(12)

It is worthwhile to point out that coma due to decentering errors depends on \( \beta \) only insofar as it is part of \( K_2 \). Different values of \( K_2 \) refer to different types of optical design. The free parameter, choosing \( m \) and \( \beta \) and subsequently \( k \), is \( K_1 \). From such set of values \( K_2 \) can be directly derived

\[
K_2 = -\left(\frac{m+1}{m-1}\right)^2 + \frac{m^3}{k(m-1)^3}(K_1 + 1)
\]

(13)

using the usual condition to get null third-order spherical aberration\textsuperscript{12}.

Interesting cases are the classical Cassegrain \((K_1 = -1)\) and Ritchey-Chretien, where

\[
\begin{align*}
K_1 &= -1 - \frac{2(1+\beta)}{m^2(m-\beta)} \\
K_2 &= -\left(\frac{m+1}{m-1}\right)^2 - \frac{2m(m+1)}{(m-\beta)(m-1)^3}
\end{align*}
\]

(14)

It can be seen that the way coefficients in eqs.(12) depend upon the optical design is rather intricated and for this reason the discussion is hereafter parametrized upon the \( K_2 \) coefficient only.

It is interesting to point out that the eqs.(12) can be put in a form analogous to eqs.(11), i.e.
\[ \text{ATC} = \left( \alpha - \frac{l}{R_o} \right) \cdot q_\alpha \] (15)

where

\[ R_o = \frac{q_\alpha}{q_l} = \frac{R_2}{1 - K_2 (\frac{m-1}{m+1})} \] (16)

and

\[ q_\alpha = \frac{3(m-1)(1+\beta)}{16F^2} \] (17)

\( R_o \) defines the so-called neutral point. A rotation around such point produces no third-order coma but almost pure tilt. This can be useful for tracking purposes, and is currently adopted in some, especially IR, telescopes\textsuperscript{13,14,15,16}.

### 3.3 Tilt term on the overall blur due to coma term

The distribution of light on the focal plane is described in a geometrical approximation by the transformation of the location, on the focal plane itself, of an incoming ray through the input pupil.

The only free parameter characterizing the blur (and not its position and orientation) is ATC. Displacements are referred to the gaussian conjugate on the focal plane, i.e. in respect to the point on which fall paraxial rays: for such a point \( \Delta \xi = 0, \Delta \eta = 0 \), and they are given\textsuperscript{17,18} by

\[ \begin{cases} 
\Delta \xi = -\frac{\text{ATC}}{3} \rho^2 \sin 2\varphi \\
\Delta \eta = -\frac{\text{ATC}}{3} \rho^2 (2 + \cos 2\varphi)
\end{cases} \] (18)

where \( \rho, \varphi \) describe the input pupil in normalized coordinates.

It is immediately possible to see that for any annular pupil the averaged displacement orthogonal to coma flare is null, i.e. \( \overline{\Delta \xi} = 0 \). For a telescope with annular pupil characterized by a linear obstruction ratio \( \varepsilon \), the barycenter of the blur due to coma is given by

\[ \overline{\Delta \eta} = \frac{-\text{ATC}}{3\pi(1-\varepsilon^2)} \int_0^{2\pi} \int_\varepsilon^1 \rho^2 (2 + \cos 2\varphi) \rho d\rho d\varphi = -\frac{\text{ATC}}{3} \left( \frac{1 - \varepsilon^4}{1 - \varepsilon^2} \right) = \gamma \cdot \text{ATC} \] (19)

Such value lies in the range from 1/3 to 2/3 of \( -\text{ATC} \) depending on the value of \( \varepsilon \), in fact

\[ \lim_{\varepsilon \to 1} \overline{\Delta \eta} = -\frac{2}{3} \text{ATC} \] (20)

This effect is summarized in Fig.2 where it is easily recognizable the displacement of the barycenter of the coma blur from the gaussian point.

In the same figure it is shown, at the same scale, the size of the circle containing 80% of the energy collected in a pure geometrical approximation. Such term is indicated, as usual, with GEE80.

A comparison between \( \overline{\Delta \eta} \) and GEE80 shows that they are of the same order of magnitude.
3.4 Derivation of the detailed tilt term effect

Summing eqs. (11) and (19), after introducing (15) in the latter gives the exact relation to the third-order for displacement $\theta^{(3)}$ of the barycenter of the blur due to M2 misalignment

$$\theta^{(3)} = \left( \alpha - \frac{l}{R_2} \right) \cdot c_\alpha + \left( \alpha - \frac{l}{R_\omega} \right) \cdot q_\alpha \gamma$$

(21)

that can be splitted, as usual, in the two parts

$$\begin{cases} 
\theta^{(3)}_a = \alpha(c + q\gamma) = \alpha \left[ \frac{2(1+\beta)}{m+1} - \frac{(m+1)(1+\beta)}{16F^2} \left( \frac{1-\epsilon^4}{1-\epsilon^2} \right) \right] = \alpha \cdot c^{(3)}_a \\
\theta^{(3)}_l = -l \left( \frac{c}{R_2} + \frac{q\gamma}{R_\omega} \right) = -l \left[ \frac{(m-1)(1+\beta)}{16F^2} \left( \frac{1-\epsilon^4}{1-\epsilon^2} \right) \right] = l \cdot c^{(3)}_l
\end{cases}$$

(22)

The interesting point is that eq.(21) can be rewritten in the form
\[ \theta^{(3)} = \left( \alpha - \frac{l}{R^*} \right) \cdot c^{(3)} \] (23)

where

\[ c^{(3)} = c_\alpha + q_\alpha \gamma \] (24)

and (omitting the subscripts in \( \alpha \))

\[ R^* = \frac{c + q\gamma}{c/R_2 + \frac{q}{R_\omega}} = R_2 \cdot \frac{1 - \frac{(m-1)(m+1)}{32F^2} \left( \frac{1-\varepsilon_t}{1-\varepsilon_s} \right)}{1 - \frac{(m-1)(m+1)}{32F^2} \left( \frac{1-\varepsilon_s}{1-\varepsilon_t} \right) \left[ 1 - K_2 \left( \frac{m-1}{m+1} \right) \right]} \] (25)

The meaning of eq.(25) is that the point around which the M2 vertex is to be rotated to give zero offset at the third-order approximation is the harmonic mean of the distance from the vertex of M2, of the center of curvature and of the neutral point, weighted respectively with the angular coefficients for the first-order effect and for the coma-displacement effect.

4 M2 behaviour from the optical point of view

The results here obtained can be summarized in a compact and elegant way (following the one given in truncated at the first order for the offset term).

It can be pointed out, in fact, that any rotation of M2 around a fixed point laterally displaced \( d \) from the optical axis can be described (see Fig.3a) by any pair of terms \( \alpha, l \) (one rotation and one displacement) related to the vertex \( V \), of the type

\[ \begin{align*}
\{ & l = d + \psi R \\
& \alpha = \psi
\end{align*} \] (26)

where \( \psi \) is to be intended as a free parameter. The term appearing in eqs.(11), (15) and (23) becomes

\[ \left( \alpha - \frac{l}{R} \right) = \psi - \frac{d + \psi R}{R} = \frac{d}{R} \] (27)

that is independent from the free parameter \( \psi \). In this way the behaviour of M2 can be fully described by the displacement of the optical axis of M2 itself from the two points defined by \( R^* \) and \( R_\omega \) (see Fig.3b). Following the prescriptions given by \( 19 \) the results can be summarized by the pair of equations

\[ \begin{align*}
\{ & ATC = d_\omega \cdot \frac{q}{R_\omega} \\
& \theta^{(3)} = d^* \cdot \frac{c^{(3)}}{R^*}
\end{align*} \] (28)

Several values for \( R^*/R_2 - 1 \) are reported in Fig.4 for a class of Ritchey-Chretien telescopes. From the graph it is evident that for typical cases, \( R^* \) differs from \( R_2 \) only by some percents.
5 A comparison with ray tracing data in the case of TNG telescope

The TNG is a Ritchey-Chretien telescope with the same optical design of the NTT (ESO).

Optical parameters of this telescope useful to our discussion are collected in Table 1. Ray-tracing verifications of eqs.(28), that in our case become

\[
\begin{align*}
\left\{
\begin{array}{l}
ATC'[^{\circ}] & \approx 0.1050 \cdot d_\omega [mm] \\
\theta^{(3)}[^{\circ}] & \approx 21.12 \cdot d^* [mm]
\end{array}
\right.
\end{align*}
\]  

(29)

were performed using our own software following prescriptions given in \textsuperscript{20}. Results are summarized in Fig.5, where it is shown the close matching between eqs.(29) and ray tracing results from pure rotations around points displaced \(d\) from the point individuated by \(R^*\).

For the TNG the difference between \(R_2\) and \(R^*\) is \(40\)mm, that can be easily taken care of in the construction of the M2 support unit.

Figure 3: a: any rotation around a point displaced \(d\) from the optical axis is determined by the parameter \(\psi\); b: decentering coma and third-order offset depend upon \(d^*\) and \(d_\omega\) respectively.
Ritchey–Chretien

\[ \epsilon = 0.33 \]

\[ \beta = 0.3767 \]

Figure 4: Differences of \( R^* \) in respect to \( R_2 \) for a class of Ritchey-Chretien telescopes.

### NTT-TNG optical parameters

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<tr>
<th>Parameter</th>
<th>Value</th>
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<td>( \epsilon )</td>
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</tr>
<tr>
<td>( R^* )</td>
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</tr>
</tbody>
</table>

Table 1: Optical parameters of NTT-Galileo useful in the text; undimensionless values are quoted in \( mm \).
Figure 5: Close matching with the expected theoretical value for $\theta^{(3)}$. Differences from theoretical values and raytracing results, normalized to ATC are here shown.

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